ASSOCIATION FOR AUTOMATED REASONING

NEWSLETTER

No. 24 November 1993

From the AAR President, Larry Wos...

After seven years, Bill McCune has resigned as AAR Secretary. This issue of the *AAR Newsletter* includes an introductory note from the association's new secretary, Bob Veroff. I encourage you to read his note and besiege him with mail—with dues, with requests for receiving the newsletter via ftp, and so on.

I am also delighted to include the announcement that Hantao Zhang has received an NSF National Young Investigator's award. As this award indicates, the field of automated reasoning is clearly gaining national attention. The AAR welcomes further announcements of this type, as well as articles designed to stimulate further research or to report on recent successes.

One such article, included at the end of this newsletter, features a set of challenge problems. Good luck in solving them!

New Secretary for AAR

Bob Veroff

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I have recently taken over from Bill McCune as secretary of AAR. I thank Bill for his efforts as secretary and for helping me become familiar with the responsibilities of the position. My main tasks as secretary will be to maintain the membership list, to beg and/or harass people to stay current with their dues, and to work with Gail Pieper to put out the AAR newsletter.

As of September 30, 1993, we have 328 members. Unfortunately, far too many members are behind in their dues. I underscore the message from the AAR president that you bring your membership up to date; the association bank account is dwindling. Dues are $7.00 for one year, $13.00 for 2 years, and $18.00 for three years. Send dues to me, payable in US dollars to the "Association for Automated Reasoning".

Remember that you now have the option of receiving the AAR newsletter by e-mail instead of on paper. We also will make AAR newsletters available by anonymous FTP. Very few people have taken us up on this offer. Send me mail if you wish the details.

We may occasionally wish to contact AAR members about articles or new procedures. We have e-mail addresses for only about 100 of the 328 members. Please send me your e-mail address if you think we may not have it and if you don't mind having it included in our membership database.

Congratulations to Hantao Zhang

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I am very happy and proud to inform you that Professor Hantao Zhang, University of Iowa, is the recipient of the NSF National Young Investigator (previously known as President's Young Investigator) award this year. As far as I know, Hantao is the first one selected for this award in the field of automated reasoning, which makes this a double honor for Hantao. And, this also reflects well for the field of automated reasoning.

As some of you may know, Hantao and I have been working together on the theorem prover RRL (Rewrite Rule Laboratory) since 1985. I was lucky to have Hantao as my student and now as a collaborator.

I wish him more such honors and awards in his illustrious career.

On the Least Herbrand Model for Conditional Horn Sets

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The least Herbrand model plays a central role in the theory of logic programming and constrained resolution. It is a well-established result [1, 2] that for every Horn set S, the intersection M_S of all Herbrand models of S is a model of S, called the least Herbrand model of S, and the members of M_S are precisely those that are logical consequences of S. In this note, we present an extension of this result to conditional Horn sets (cHs).

We shall divide the entire set of atoms of the first-order language under this discussion into two disjoint subsets, namely, R-atoms and K-atoms. Let R-Hb be the set of ground atoms of R-atoms and let K-Hb be the set of ground atoms of K-atoms. Then R-Hb $\cup$ K-Hb is the Herbrand base of the language, and each Herbrand interpretation is a subset of R-Hb $\cup$ K-Hb. A set S is called a *conditional Horn set* (cHs) wrt R-atoms iff the restriction of S on R-atoms, $S|_{R-atoms}$, is a Horn set. According to this definition, we shall write each clause of a cHs in a form of $D/H/R$, where D is a member of R-atoms, H is a conjunction of literals, whose atoms are members of K-atoms, and R is a conjunction of members of R-atoms. An ordinary Horn set S can be viewed as a special cHs, for which $S|_{R-atoms}$ is identical to S. The concept cHs is a nontrivial extension of Horn set because a cHs as the whole may not be a Horn set and may not be able to be transformed into a Horn set. For a real application, a (non-Horn) cHs may be needed for accurately specifying the typing structure of a program, which contains predicate subtypes (i.e., $x : nat/0 \leq x/x : int$) or partial functions (i.e., $div(x, y) : int/y \neq 0/x : int \wedge y : int$).

The problem under study here is as follows: given a cHs S, determine whether S has a least Herbrand model in a sense similar to that for Horn set. Stated more precisely, Let S be a cHs, B a subset of K-Hb, and M a subset of R-Hb. M is called a B-based *conditional Herbrand model* (cHm) of S iff $B \cup M$ is a Herbrand model of S. M is called a least B-based cHm of S iff M is a B-based cHm of S, which equals the intersection of all B-based cHms of S. Our question is: for every $B \subset K$-Hb, if such a least B-based cHm exists, how can it be constructed, what properties does it possess, and what usage may it have?

Recall that for a Horn set S (on R-atoms), the least Herbrand model of S can be constructed as a set $\{d \in R\text{-}Hb \mid S \models d\}$. By a straightforward extension of this result, one may conjecture that $M \equiv \{d \in R\text{-}Hb \mid B \cup S \models d\}$ is the least B-based cHm. However, this conjecture is incorrect. Consider the cHs $S_0 \equiv \{d_0/\neg p_0\}$ wrt R-atoms. (Assume for $i \equiv 0, 1, ..., d_i \in R$-Hb, $p_i \in K$-Hb.) Let B_0 be the empty set ϕ. Since none of atoms of R-Hb is a consequence of $B_0 \cup S_0$, M will be ϕ by the conjecture. But ϕ is not a B_0-based cHm of S_0, because $B_0 \cup \phi \equiv \phi$, and ϕ is not a model of S_0.

Definition 1. Let S be a cHs wrt R-atoms, and let r_0 be a conjunction on R-Hb. Let $(S)_{gr}$ be the entire set of ground instances of clauses of S. Define the set of *ground Horn preconditions* of r_0 in S by

$$gHp(S, r_0) \equiv \left\{ h_1 \wedge ... \wedge h_n \;\middle|\; \begin{array}{l} \{d_1/h_1/r_1, ..., d_n/h_n/r_n\} \subset (S)_{gr} \text{ and} \\ \{d_1//r_1, ..., d_n//r_n, \neg r_0\} \text{ is mini-unsatisfiable} \end{array} \right\},$$

and define the *weakest Horn precondition* (wHp) of r_0 in S by

$$wHp(S, r_0) \equiv \bigvee_{W \in gHp(S, r_0)} (W).$$

Intuitively, a member of $gHp(S, r_0)$ is a condition under which r_0 is implied by S, and $wHp(S, r_0)$ is the weakest condition under which r_0 is implied by S. (We use $true$ for empty clause, $false$ for empty literal, $D/true/R$ for $D//R$, and D/H for $D/H/true$.)

Example 1. For $S_0 \equiv \{d_0/\neg p_0\}$, $gHp(S_0, d_0) \equiv \{\neg p_0\}$, and $wHp(S_0, d_0) \equiv \neg p_0$.

Example 2. Let $S_1 \equiv \{d_1, d_2/p_1, d_3/p_1 \wedge p_2/d_1, d_4/\neg p_1 \wedge \neg p_2, d_4/p_3/d_1, d_6//d_5\}$.

$$\begin{array}{ll} gHp(S_1, d_1) \equiv \{true\}, & wHp(S_1, d_1) \equiv true, \\ gHp(S_1, d_2) \equiv \{p_1\}, & wHp(S_1, d_2) \equiv p_1, \\ gHp(S_1, d_3) \equiv \{p_1 \wedge p_2\}, & wHp(S_1, d_3) \equiv p_1 \wedge p_2, \\ gHp(S_1, d_4) \equiv \{\neg p_1 \wedge \neg p_2, p_3\}, & wHp(S_1, d_4) \equiv (\neg p_1 \wedge \neg p_2) \vee p_3, \\ gHp(S_1, d_6) \equiv \phi, & wHp(S_1, d_6) \equiv false. \end{array}$$

Lemma 1. Let S be a cHs wrt R-atoms, d_j an element of R-Hb, and B a subset of K-Hb. If B is a Herbrand model of $wHp(S, d_j)$ (written by $[wHp(S, d_j)]^B \equiv true$), then for every B-based cHm M of S, $d_j \in M$.

Proof. From Definition 1 and the fact $[wHp(S, d_j)]^B \equiv true$, there exists at least one element g_j of $gHp(S, d_j)$, such that $[g_j]^B \equiv true$. Then, there exists a set, $S_j \equiv \{d_{j_1}/h_{j_1}/r_{j_1}, ..., d_{j_k}/h_{j_k}/r_{j_k}\} \subset$

$(S)_{gr}$, such that $g_j \equiv h_{j_1} \wedge ... \wedge h_{j_k}$, and $S_j|_{R\text{-atoms}} \cup \{\neg d_j\}$ is unsatisfiable. Let $d_{j_i}/h_{j_i}/r_{j_i}$ be an arbitrary element of S_j. Since M is a B-based cHm of S, $[d_{j_i}/h_{j_i}/r_{j_i}]^{B \cup M} \equiv true$. Then since $[h_{j_i}]^B \equiv true$, $[d_{j_i}//r_{j_i}]^{B \cup M} \equiv true$. Furthermore, since B shares no atoms with d_{j_i} and r_{j_i}, $[d_{j_i}//r_{j_i}]^M \equiv true$. Thus we conclude that M is a model of $S_j|_{R\text{-atoms}}$. However, $S_j|_{R\text{-atoms}} \cup \{\neg d_j\}$ is unsatisfiable. Then $\neg d_j$ must be false in M, and consequently, $d_j \in M$. Q.E.D.

The least B-based cHm for a conditional Horn set is defined in the following theorem.

Theorem 1 (*Least Herbrand Model Theorem for conditional Horn sets*). Let S be a cHs wrt R-atoms, and let B be a subset of K-Hb. Define

$$M_S^B \equiv \{d \in R\text{–Hb} \mid [wHp(S,d)]^B \equiv true\}.$$

Then M_S^B must be a B-based cHm of S, and M_S^B equals the intersection of all B-based cHms of S. Accordingly, we call M_S^B the *least B-based cHm* of S.

Example 3. From Example 1 we have that $wHp(S_0, d_0) \equiv \neg p_0$. For $B_0 \equiv \phi$, noting that $[p_0]^{B_0} \equiv false$, and so $[wHp(S_0, d_0)]^{B_0} \equiv true$, we conclude

$$M_{S_0}^{B_0} \equiv \{d_0\}.$$

It is easy to verify that, $d_0/\neg p_0$ is true in $B_0 \cup M_{S_0}^{B_0}$, and so $M_{S_0}^{B_0}$ is indeed a B_0-based cHm of S_0. Note also that, since B_0 is empty, each B_0-based cHm of S_0 must have d_0 as a member. Then $M_{B_0}^{S_0}$ must be equal to the intersection of all B_0-based cHms of S_0.

Example 4. For the set S_1 given earlier, we present the B-based least cHm of S_1 for different subset B of K-Hb in the following table.

	B	$M_{S_1}^B$	$B \cup M_{S_1}^B$	$[S_1]^{B \cup M_{S_1}^B}$
1.	$\{\}$	$\{d_1, d_4\}$	$\{d_1, d_4\}$	true
2.	$\{p_1\}$	$\{d_1, d_2\}$	$\{p_1, d_1, d_2\}$	true
3.	$\{p_1, p_2\}$	$\{d_1, d_2, d_3\}$	$\{p_1, p_2, d_1, d_2, d_3\}$	true
4.	$\{p_3\}$	$\{d_1, d_4\}$	$\{p_3, d_1, d_4\}$	true
5.	$\{p_1, p_2, p_3\}$	$\{d_1, d_2, d_3, d_4\}$	$\{p_1, p_2, p_3, d_1, d_2, d_3, d_4\}$	true

Proof. We now prove Theorem 1 by the following two steps.

Step 1. We first prove that M_S^B given in Theorem 1 is a B-based cHm of S. Assume that M_S^B is not a B-based cHm of S. We are going to deduce a contradiction from this assumption. Let $I \equiv B \cup M_S^B$, then there exists at least one member $d_0/h/d_1 \wedge ... \wedge d_n$ of $(S)_{gr}$ which is false in I. Hence we must have $[d_0]^I \equiv false$, $[h]^I \equiv true$, and for every $j, 1 \leq j \leq n$, $[d_j]^I \equiv true$. Since $d_j \notin B$, $[d_j]^I \equiv true$ implies that $[d_j]^{M_S^B} \equiv true$. Then from the definition for M_S^B, $[wHp(S, d_j)]^B \equiv true$. From Definition 1, there exists at least one element g_j of $gHp(S, d_j)$ which is true in B. Consequently, there exists a set $S_j \subset (S)_{gr}$, $S_j \equiv \{d_{j_1}/h_{j_1}/r_{j_1}, ..., d_{j_k}/h_{j_k}/r_{j_k}\}$, such that $g_j \equiv h_{j_1} \wedge ... \wedge h_{j_k}$, and $S_j|_{R\text{-atoms}} \cup \{\neg d_j\}$ is mini-unsatisfiable. Let $S_0 \equiv S_1 \cup ... \cup S_n \cup \{d_0/h/d_1 \wedge ... \wedge d_n\}$. $S_0|_{R\text{-atoms}} \cup \{\neg d_0\}$ must be unsatisfiable. Let $S_0' \equiv \{d^1/h^1/r^1, ..., d^s/h^s/r^s\}$ be a subset of S_0, such that $S_0'|_{R-\text{atoms}} \cup \{\neg d_0\}$ is mini-unsatisfiable. Then $g' \equiv h^1 \wedge ... \wedge h^s$ must be a member of $gHp(S, d_0)$, and hence a disjunct of $wHp(S, d_0)$. However, each conjunct h^l of g' must be either h (if h^l comes from $d_0/h/d_1 \wedge ... \wedge d_n$) or a conjunct h_{j_i} of g_j for some $j, 1 \le j \le n, j_1 \le j_i \le j_k$ (if h^l comes from an element of S_j). For each case, it is easy to determine $[h]^B \equiv true$, and so we conclude that $[g']^B \equiv true$. Since g' is a disjunct of $wHp(S, d_0)$, $[wHp(S, d_0)]^B \equiv true$. Consequently, $d_0 \in M_B^S$, and $[d_0]^I \equiv true$. This result contradicts the earlier assertion that $[d_0]^I \equiv false$.

Step 2. We now prove that M_S^B equals the intersection of all B-based cHms of S. With the assertion proved in Step 1, we need only to prove a conjecture that for every B-based cHm M of S, $M_S^B \subset M$. But this conjecture follows directly Lemma 1. Q.E.D.

Proposition 1 given below states that Theorem 1 is a generalization of the classical least Herbrand model theorem for Horn sets. Proposition 2 states a relation between the least B-based cHm of a cHs S and the least Herbrand model of the Horn set $S|_{R-\text{atoms}}$.

Proposition 1. If S is a Horn set on R-atoms, then for every subset B of K-Hb, M_S^B defined in Theorem 1 is the ordinary least Herbrand model of S.

Proof. Since S is a Horn set on R-atoms, each member of S must be of form $D/true/R$. Then for every element d of R-Hb, if $S \models d$, then $gHp(S, d)$ must be the set $\{true\}$, and so $wHp(S, d) \equiv true$. If $S \not\models d$, then $gHp(S, d)$ must be the set ϕ, and so $wHp(S, d) \equiv false$. Thus, the condition $[wHp(S, d)]^B \equiv true$ for d being a member of M_S^B is equivalent to the condition that d is a logical consequence of S. Q.E.D.

Proposition 2. Let S be a cHm wrt R-atoms and let M_0 be the least Herbrand model of $S|_{R-\text{atoms}}$. For every $B \subset K$-Hb, M_S^B must be a subset of M_0.

Proof. Trivial. Q.E.D.

The least Herbrand model theorem for conditional Horn sets was used by the author in proving the completeness of some specialized proof procedures for deduction with constrained theories which are conditional Horn sets.

References

1. M. H. van Emden and R. A. Kowalski, "The semantics of predicate logic as a programming language," *J. ACM* 23(4) (Oct. 1976), 733–742.

2. J. W. Lloyd, *Foundations of Logic Programming,* Springer-Verlag, New York (1984).

New Journal Announced

We have received an announcement that a new journal, entitled the *Electronic Journal of Functional & Logic Programming* (EJFLP), will begin publication in late 1993. The journal is fully refereed and will be available via e-mail free of charge. For subscriptions, send an empty message to subscriptions@ls5.informatik.uni-dortmund.de.

Papers are solicited in functional and logic languages; integration of programming paradigms; parallelism in functional and logic programming; program interpretation, compilation, and transformation; static analysis; semantic foundations; proof calculi for functional, logic, and constraint programming; applications; and declarative programming concepts and methodology. Papers should be submitted in postscript or dvi format to submissions@ls5.informatik.uni-dortmund.de. E-mail an empty message with "Help" in the subject field for submission information.

Members of the EJFLP Editorial Board include R. Loogen, H. Kuchen, M. Hanus, M. MT Chakravarty, M. Koehler, Y. Guo, M. Rodriguez-Artalejo, A. Krall, A. Mueck, T. Ida, H. C. R. Lock, A. Hallmann, P. Padawitz, C. Brzoska, and F. Pfennig.

New Center for Basic Research in Computer Science

A new Center for Basic Research in Computer Science (BRICS) will be established at Aarhus University, in association with Aalborg University, Denmark. The center is to begin in January 1994 for a duration of at least five years. Yearly support of about 1 million US$ will be provided by the Danish National Research Foundation.

The center will encourage research in logic, algorithmics, and semantics and their interplay. It will fund visiting and postdoctoral positions and graduate student scholarships, and will organize seminars and meetings. For further information, write to Glynn Winskel or Uffe Engberg (attn. BRICS), Computer Science Department, Aarhus University, Ny Munkegade, Bldg. 540, DK-8000 Aarhus C, Denmark. E-mail gwinskel@daimi.aau.dk or Fax: +45 8613 5725.

Call for Papers

Artificial Intelligence and Mathematics

The 3rd International Symposium on Artificial Intelligence and Mathematics will be held in Fort Lauderdale, Florida, on January 2–5, 1994. This is a biennial series featuring applications of mathematics in artificial intelligence as well as artificial intelligence techniques and results in mathematics. For further information, write to Frederick Hoffman, Department of Mathematics, Florida Atlantic University, P.O. Box 3091, Boca Raton, FL 33431 (hoffman@acc.fau.edu).

International Conference on Logic Programming

The 1994 International Conference on Logic Programming will take place in Santa Margherita Ligure, Italy, June 13–18, 1994. The deadline for submitting papers is November 15, 1993. Topics include all theoretical and practical aspects of logic programming. For further information, send e-mail to martelli@disi.unige.it or pvh@cs.brown.edu.

CADE-12

As we announced in the last AAR Newsletter, the Twelfth International Conference on Automated Deduction will take place on June 28–July 1, 1994, in Nancy, France.

CADE conferences cover all aspects of automated deduction, including first- vs. higher-order logics, classical vs. nonclassical logics, special vs. general-purpose inference, and interactive vs. automatic systems. Specific topics of interest include resolution, sequent calculus, decision procedures, unification, rewrite rules, and mathematical induction. Also of interest are applications of automated deduction, including deductive databases, logic and functional programming, commonsense reasoning, and software and hardware development.

Papers should not exceed 15 proceedings pages; system descriptions and problem sets should not exceed 5 proceedings pages. Springer style files should be used if possible; send an e-mail with contents "HELP" to `svserv@dhdspri6.bitnet`; or FTP anonymously from `dream.dai.ed.ac.uk` (see instructions in `pub/cade-12/README`). The title page of the submission should include the name, address (with email if possible) and telephone number of each author. Papers must be unpublished and not submitted for publication elsewhere. Submissions that are late or too long or that require major revision will not be considered.

Authors should send 4 copies of their submission to the Programme Chair Alan Bundy, Department of Artificial Intelligence, University of Edinburgh, 80 South Bridge, Edinburgh EH1 1HN, Scotland (Tel: [+44] 31-650-2716; Fax: [+44] 31-650-6516)

Submission deadline:	December 1, 1993
Notification of acceptance:	February 14, 1994
Camera-ready copy due:	March 29, 1994

Logic in Computer Science

The Ninth Annual IEEE Symposium on Logic in Computer Science (LICS) The LICS symposia aim to attract high-quality original papers covering theoretical and practical issues in computer science that relate to logic in a broad sense, including algebraic, categorical and topological approaches. Topics of interest to AAR members include automated deduction, knowledge representation, lambda and combinatory calculi, logical aspects of computational complexity, logics in artificial intelligence, logic programming, modal and temporal logics, program logic and semantics, rewrite rules, logical aspects of symbolic computing, and verification.

Submission requirements are as follows: 10 hard copies of a detailed abstract (not to exceed 10 typed pages) and 20 additional copies of the cover page should be received by December 13, 1993, by the program chair: Samson Abramsky, Attn: LICS, Department of Computing, Imperial College, 180 Queen's Gate, London SW7 2BZ, United Kingdom (e-mail sa@doc.ic.ac.uk; phone (44) 71-589-5111 ext. 5005; Fax: (44) 71-581-8024). The cover page of the submission should include the title, authors, a brief synopsis, and the corresponding author's name, address, phone number, fax number, and e-mail address, when available. Abstracts must be in English, clearly written, and provide sufficient detail to allow the program committee to assess the merits of the paper. References and comparisons with related work should be included. It is recommended that each submission begin with a succinct statement of the issues, a summary of the main results, and a brief explanation of their significance and relevance to the conference, all phrased for the non-specialist. Technical development of the work, directed to the specialist, should follow.

Problem Set

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My students have found the following set of problems to be both challenging and enlightening while learning to use OTTER. Several people have asked me for this set, so I have decided to include it in an AAR newsletter.

Consider the following definition of a Boolean algebra: A Boolean algebra is a tuple $(S,+,*,',0,1)$, where S is a nonempty set of elements, + and * are binary operations on S, ' is a unary operation on S, and 0 and 1 are distinct elements of S and the following axioms hold:

For all x, y, and z in S,

A1. Commutativity: $x + y = y + x$; $x * y = y * x$.

A2. Distributivity: $x + (y * z) = (x + y) * (x + z)$; $x * (y + z) = (x * y) + (x * z)$.

A3. Identity: $x + 0 = x$; $x * 1 = x$.

A4. Complement: $x + x' = 1$; $x * x' = 0$.

Prove the following theorems.

For all x, y, and z in S,

TA. Idempotence: $x + x = x$; $x * x = x$.

TB. Boundedness: $x + 1 = 1$; $x * 0 = 0$.

TC. Absorption: $x + (x * y) = x$; $x * (x + y) = x$.

TD. Associativity: $x + (y + z) = (x + y) + z$; $x * (y * z) = (x * y) * z$.

TE. Uniqueness of Complement: if $x + y = 1$ and $x * y = 0$, then $y = x'$.

TF. Involution: $(x')' = x$.

TG. Complement of 0 and 1: $0' = 1$; $1' = 0$.

TH. DeMorgan's Laws: $(x + y)' = x' * y'$; $(x * y)' = x' + y'$.